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A Robust Integrated Model for Resource-Constrained Project Scheduling Problem with Material Ordering

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Abstract: Nowadays, the importance of careful planning to achieve goals in project-based organizations (PBOs) has become clear with the complexity of economic conditions and competitive conditions governing the receipt and implementation of projects. One of the most important of these plans is project scheduling, which is an attractive research field for the use of optimization methods among researchers in operation research due to its importance. This study was conducted to investigate the problem of robust integrated project scheduling and material ordering in terms of resource constraints, uncertainty, and lag times. Simultaneous consideration of discount, non-renewability, perishability, uncertainty, and different project implementation scenarios brought the model closer to real-world conditions and, as a result, made the results more practical. So, a scenario-based model was designed with the objective function to minimize the total cost (including ordering, maintenance, purchasing, and penalties for delay minus the bonus for hastening the project delivery). Model robustification was then performed with a possibilistic approach. After providing the model, numerical problems in different dimensions were designed and solved by using GAMS software, and the results were discussed.

Keywords: robust model, material ordering, scheduling, uncertainty, discount, perishability.

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1. Introduction

With the passage of time, increasing the volume of projects, and diversifying the tools which used to complete a project, the need to change project scheduling methods from traditional to scientific are well felt in such a way that the limited and available valuable resources are used to achieve the project goals that lie in three sides of the project triangle, namely time, cost, and quality (Mubarak, 2015). Classical approaches such as the critical chain assume that project resources are unlimited. However, this assumption does not make sense in most projects. Subsequently, the resource constraint assumption was added to the model. Such problems are called resource-constrained project scheduling problems (RCPSP) (Liu *et al.*, 2017).

Project management issues such as scheduling and material supply-related decisions are interrelated, and a separate island look at these issues can reduce the model's accuracy and applicability. Even the best schedules cannot be very useful for the project if the ordering and supply of required resources are disrupted (Tabrizi and Ghaderi, 2016). Hence, the need to design a model that presents decisions about scheduling and resource provisioning in the form of an integrated and simultaneous model is strongly felt. For this reason, integrated models of project scheduling and material ordering have attracted the attention of researchers in recent years (Artigues *et al.*, 2013).

Integrating the project scheduling problem with material ordering was first introduced by Aquilano and Smith (Aquilano and Smith, 1980). They developed an integrated model including material requirements planning such as material list, material waiting time, inventory level scheduling, and critical path method. Since then, several studies have been conducted in this field, some of which are discussed below. Ke *et al*. provided an uncertain model for the resource-constrained project scheduling problem (RCPSP) and modeled it with a robust approach (Ke *et al.,* 2015). They considered the schedule of activities in the project logistics with uncertainty. In a study by Tabrizi and Ghaderi, a model with a price discount was presented, taking into account the objectives of maximizing model robustness and minimizing project costs (Tabrizi and Ghaderi, 2016). Zoraghi *et al.* modeled the same problem with the three objectives of minimizing project completion time, maximizing project robustness, and minimizing activity completion time (Zoraghi *et al.*, 2017). In a study by Tabrizi, previous models were upgraded by considering environmental constraints and considered it as one of the objective functions of the problem (Tabrizi, 2018). Habibi *et al.* presented a model for project scheduling and material ordering, assuming discounts and robustness indicators. The model had three objectives of maximizing the net present value (NPV) of project cash flows, maximizing project scores in terms of social indicators, and maximizing project scores in terms of environmental indicators (Habibi *et al.*, 2019).

In real conditions, ignoring some limitations and realities in modeling can reduce the applicability of the proposed model. Therefore, in this study, several important issues in modeling are considered. These issues include non-renewability, perishability, discount project resources, uncertainty, lag times, and equipment disruption. The following is a brief description of these concepts for more familiarity with them.

In most studies assume that activities are performed in an ideal setting and the proposed schedule can be executed exactly according to the plan (Mubarak, 2015). The existence of uncontrollable factors such as lack of access to resources, the addition of unforeseen activities to the project, and bad weather conditions practically lead to the failure to achieve the project objectives in the desired time. This can bring significant costs to the project (Artigues *et al.*, 2013). Therefore, one of the main challenges facing construction projects is the existence of uncontrollable factors. The effect of these factors on the project can be greatly reduced if different scenarios are predicted in the planning done for the project and the planning is done based on these scenarios (Ke *et al.*, 2015). Robust optimization is a new approach that has been proposed in recent years to deal with data uncertainty in various scenarios. In this approach, near-optimal solutions are considered which are highly feasible and resistant to change. In other words, the feasibility of the solution obtained in different scenarios is guaranteed by slightly deviating from the value of the objective function (Ben-Tal *et al.*, 2009). Accordingly, in this study, a robust optimization approach is used to deal with changes in different scenarios to minimize the effect of different modes of events in the project on the accuracy of the plans made.

There are many different types of resources for project scheduling and control. Two main types of these resources are renewable and non-renewable (Demeulemeester and Herroelen, 2006). Non-renewable resources are those that, when an activity is completed, the amount of resources allocated to it is exhausted and that specified amount can no longer be reused. Gypsum, cement, and materials used in the project can be mentioned among these resources (Kerzner, 2017). One of the non-renewable resources is perishable resources. Perishable resources are those resources that have a certain period to consume and cannot be consumed, or their quality gradually decreases if they are not consumed by the specified time. For example, mix concrete for construction projects is a perishable resource. Non-perishable resources are resources that do not have a specific expiration date and their quality does not change over time (Tabrizi and Ghaderi, 2016).

Another fact of business is discount. Some sellers offer their buyers lower prices than the standard price for buying with a certain volume. For example, if the price of a commodity is p, the seller offers the price of each commodity p-x (a positive number) in exchange for buying more than n of that commodity in the hope that the commodity will be sold more and buyers will be more willing to buy the commodity (Tabrizi and Ghaderi, 2016). Since construction projects often require large quantities of consumables, contractors and project managers can subject the project to total or incremental discounts by planning appropriate quantities of materials to purchase. So, in planning projects with limited resources, one of the important decisions that must be made by managers is to buy materials in what periods and in what quantities so that the project can benefit from the discounts offered by the suppliers in addition to meeting the project needs for resources (Tabrizi, 2018).

Another problem with the real world is the uncertainty in the parameters of the problem. In the real world, some parameters of the problem are uncertain. Failure to pay attention to this uncertainty will reduce the validity of the model. Uncertainty in construction projects plays a significant role and affects the planning of project managers (Ma *et al.*, 2015). Some of the most important effects of uncertainty on construction projects are changes in material prices, changes in project costs, changes in the timing of activities compared to what was planned, etc. (Bruni *et al.*, 2018). Traditional approaches to dealing with uncertainty require additional data such as probability distribution and membership function, and it is difficult to work with these approaches due to the high volume of data required. Therefore, applying other theories can be helpful in this way (Bertsimas *et al.*, 2011).

Another issue that should be considered in modeling the project scheduling problem, so that the proposed schedule is most in line with real-world conditions, is the lag times such as public holidays and occasions in which the contractor is required to close the project.

According to the above, the study seeks to provide a robust integrated model for the resource-constrained project scheduling problem (RCPSP), taking into account material supply constraints. Considering issues and realities such as lag times, uncertainties, discounts, and various project scenarios, the integrated model of the project scheduling and material ordering (PSMO) problem is expected to be more in line with the realities of the real world and its outputs will be useful for people who deal with project planning and scheduling.

The rest of the study is organized as follows: In the second part, the model is provided. The third section is devoted to solving numerical problems, and the fourth section presents conclusions.

2. Modeling

In this section, the mathematical programming model for the study problem is described. So, the problem is explained first. The model components are then introduced. Finally, the model and its components are explained.

2.1 Problem Statement

In this study, the resource-constrained project scheduling problem (RCPSP) and material ordering problem are simultaneously investigated. The resources required in the project are divided into two types: renewable and non-renewable. Renewable resources such as machinery and manpower are simply occupied for activities, which means that they are released after the activity and can be used for other activities. Non-renewable resources such as building materials are used up during activities and can no longer be used for other activities. This type of resource is also referred to as consumer resources. They have an expiration date, meaning that they are perished and can no longer be used after that period.

There are several suppliers of resources for the project. Each supplier can supply one item from these resources and offer discounts as a total discount for different amounts of purchasing.

The problem is studied in an uncertain environment, meaning that the costs of material ordering and the duration of each activity are considered as uncertain parameters in addition to the purchase prices of materials. According to Chakrabortty, the implementation of activities is divided into separate scenarios with a specific probability of occurrence due to the existence of unpredictable factors such as weather conditions and machine failure (Chakrabortty, 2017). In each scenario, the duration of each activity is certainly determined. Moreover, the purchase prices and cost of material orders are determined by uncertain trapezoidal fuzzy numbers.

The problem assumptions for modeling are as follows:

- The upper limit of the project completion time is specified, and any delay (hastening) in the delivery of the project will be subject to a penalty (bonus).

- Several scenarios with a specific probability of occurrence are considered for the time of each activity due to unpredictable reasons such as weather conditions and machine failure.
- The unit purchase price of each consumable material in each of the discount intervals is indicated by trapezoidal fuzzy numbers.
- The ordering cost of each consumable material is determined by trapezoidal fuzzy numbers.
- The available amount of each renewable resource in each period does not exceed the upper limit.
- Every activity is allowed to start only when all the resources (renewable or non-renewable) are available.
- Interruption of any of the activities is not permitted.
- Lag times are included in the model as activities with a specific length of time and a zero-resource requirement.
- Material ordering is permitted only once for each activity.
- Purchasing any consumables from any supplier has its total discount function.
- Consumables have an expiration date so that they are perished and can no longer be used after that date.
- Lag times can be included in the model as activities with a defined length of time and a zero-resource requirement.
- Disruptions in facilities, weather conditions, and other environmental factors can be considered as factors forming uncertainty scenarios, which ultimately create scenarios with a probable occurrence in such a way that the duration of each activity is specified in each scenario.

2.2 Model components

Notations

In this section, the problem notations are introduced.

$i, j \in \{0,1,2,...,n+1\}$: Project activities (0 and $n+1$ indicate the start and end of the project)

$l \in \{1,2,3,...,L\}$: Renewable resources

$f \in \{1,2,3,...,\mathrm{F}\}$: Non-renewable resources (consumables)

$t,\tau \in \{1,2,3,...,\mathrm{T}\}$: Project execution periods (T is obtained by considering the execution time of critical path activities equal to the maximum execution time)

$\mathrm{s} \in \{1,2,3,...,S\}$: Suppliers of consumables

$m \in \{1,2,3,...,M\}$: Scenarios for the duration of activities

Parameters

In this section, the parameters used in the model are introduced.

K_{fs}: The number of discount levels provided by supplier s for consumables f

$pr(i)$: Precedences (Prerequisites) for activity i

DT : Project delivery time

d_i^m : The duration of activity i in case of scenario m

r_{il} : The amount of renewable resource l required for the activity i for each period of the activity

u_{if} : The amount of consumed resource f required to perform activity i

$R_l^{\max}$: The maximum amount of renewable resource l available in each period

$\tilde{A}_{fs}$: The cost of ordering consumed resource f from supplier s

$\tilde{H}_f$: The cost of maintaining consumed resource f per period

P : The penalty for delay in project delivery per period

UC : The penalty for violating precedence restrictions by one unit of time

B : The bonus for hastening project delivery per period

$\tilde{C}_{fks}$: The cost of purchasing a unit of consumed resource f at discount level k from supplier s

γ_{fks} : The upper limit of the order amount of the consumed resource f at discount level k from supplier s

EX_f : The consumption period of consumed resource f

$\Pr ob(m)$: The probability of occurrence of any of the scenarios describing the time of activities

Variables

In this section, model variables are introduced.

TC : The total cost of the project

Δ_{ij}^{m} : The non-negative variable indicating the degree of violation of the precedence relationship between the activity pair i and j when the duration of the activity i is equal to scenario m

Y_{fkst} : The binary variable is 1 when consumed resource f is ordered at discount level k from supplier s in period t

Z_{fksit} : The binary variable is 1 when consumed resource f is ordered at discount level k from supplier s in period t and for activity

i X_{it} : The binary variable is when activity i starts in period t

2.3 The certain model of the problem

The mathematical model of the problem in the definite mode is as described in Equations 1-8.

$$Min\ TC = \sum_f \sum_s \sum_t \sum_{k=1}^{K_{fs}} \tilde{A}_{fs} Y_{fkst} + \sum_t \sum_f \sum_i \tilde{H}_f \left[\sum_{\tau=t-EX_f}^{t} \sum_s \sum_{k=1}^{K_{fs}} Z_{fksi\tau} - u_{if} \sum_{\tau=1}^{t} X_{i\tau} \right] + \sum_t \sum_f \sum_s \sum_i \sum_{k=1}^{K_{fs}} \tilde{C}_{fks} u_{if} Z_{fksit} \tag{1}$$

$$+ \sum_{t=DT+1}^{T} P\left(t-DT\right) X_{(n+1)t} + UC \sum_m \sum_i \sum_j \Pr ob\left(m\right) \Delta_{ij}^{m} - \sum_{t=1}^{DT-1} B\left(DT-t\right) X_{(n+1)t}$$

$$\sum_t X_{it} = 1 \quad ;\forall i \tag{2}$$

$$\sum_t \left(d_i^m - t\right) X_{it} - \Delta_{ij}^{m} \le \sum_t t X_{jt} \quad ;\forall j, m, \forall i \in \Pr\left(j\right) \tag{3}$$

$$\sum_i \sum_t r_{il} X_{it} \le R_l^{\max} \quad ;\forall l \tag{4}$$

$$\gamma_{f(k-1)s} Y_{fkst} \le \sum_i u_{if} Z_{fksit} \le \gamma_{fks} Y_{fkst} \quad ;\forall f, s, t, \forall k \le K_{fs} \tag{5}$$

$$\sum_{k=1}^{K_{fs}} Y_{fkst} \leq 1 \quad ;\forall f,s,t \tag{6}$$

$$X_{it} \leq \sum_{\tau=t-EX_f}^{t} \sum_{s} \sum_{k=1}^{K_{fs}} Z_{fksit} \quad ;\forall f,i,t \tag{7}$$

$$\begin{aligned} & Y_{fkst}, Z_{fksit}, X_{it} \in \{0 \; or \; 1\} \; ;\forall i,f,k,s,t \\ & \Delta_{ij}^{m} \geq 0 \quad ;\forall i,j,m \end{aligned} \tag{8}$$

Equation 1 represents the objective function of the problem and is equal to the sum of the costs (ordering, maintenance, purchase, and penalties for delays in the project delivery) minus the bonus for hastening the project delivery. Equation 2 represents the performance of all project activities. Equation 3 ensures compliance with precedence relationships. Equation 4 indicates compliance with the maximum permitted use of renewable resources. Equations 5 and 6 ensure that each consumed resource is ordered at only one of the discount levels. Equation 7 makes the performance of any activity conditional on the provision of resources for that activity. Equation 8 indicated the range of values of the variables.

2.4 Model Robustification

Among Equations 1-8, only Equation 1 has uncertain trapezoidal fuzzy parameters. So, just paying attention to this equation is sufficient for robustification. One of the most important approaches to robustifying scenario models is the approach proposed by Mulvey (Mulvey *et al.*, 1995). This approach is adopted when the occurrence scenarios are not very common, but in this case, more than half of the activities between two or more scenarios may have the same occurrence time. In other words, until the last activity of each scenario is performed, the type of scenario cannot be determined and, consequently, no systematic planning can be presented. As a result, it is not possible to use the approach proposed by Mulvey in practice (Mulvey *et al.*, 1995).

Assume that an uncertain parameter such as $\xi = (\xi(1), \xi(2), \xi(3), \xi(4))$ is a trapezoidal fuzzy number. The probability membership function ($\mu(\xi)$) of this parameter can be seen in Figure 1 (Pishvaee *et al.*, 2012). It should be noted that this function indicates the probability of occurrence of any of the parameter values.

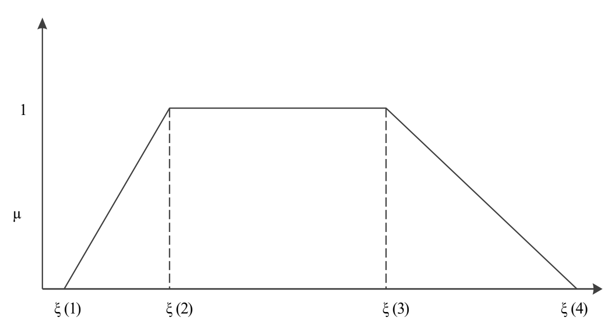

Figure 1. The probability membership function of a trapezoidal fuzzy number

According to Pishvaee *et al.*, the robust probability equivalent of Equation 1 is as described in Equation 9 (Pishvaee *et al.*, 2012).

$$Min\ TC = Avg + \gamma\left(TC^{\max} - TC^{\min}\right)$$

$$Avg = \sum_{f}\sum_{s}\sum_{t}\sum_{k=1}^{K_{fs}}\left(\frac{A^1_{fs}+A^2_{fs}+A^3_{fs}+A^4_{fs}}{4}\right)Y_{fkst} + \sum_{t}\sum_{f}\sum_{i}\left(\frac{H^1_f+H^2_f+H^3_f+H^4_f}{4}\right)\left[\sum_{\tau=t-EX_f}^{t}\sum_{s}\sum_{k=1}^{K_{fs}} Z_{fksi\tau} - u_{if}\sum_{\tau=1}^{t} X_{i\tau}\right]$$

$$+\sum_{t}\sum_{f}\sum_{s}\sum_{i}\sum_{k=1}^{K_{fs}}\left(\frac{C^1_{fks}+C^2_{fks}+C^3_{fks}+C^4_{fks}}{4}\right)u_{if}Z_{fksit} + \sum_{t=DT+1}^{T} P(t-DT)X_{(n+1)t} + UC\sum_{m}\sum_{i}\sum_{j}\Pr ob(m)\Delta^m_{ij} - \sum_{t=1}^{DT-1} B(DT-t)X_{(n+1)t} \quad (9)$$

$$TC^{\max} = \sum_{f}\sum_{s}\sum_{t}\sum_{k=1}^{K_{fs}} A^4_{fs}Y_{fkst} + \sum_{t}\sum_{f}\sum_{i} H^4_f\left[\sum_{\tau=t-EX_f}^{t}\sum_{s}\sum_{k=1}^{K_{fs}} Z_{fksi\tau} - u_{if}\sum_{\tau=1}^{t} X_{i\tau}\right] + \sum_{t}\sum_{f}\sum_{s}\sum_{i}\sum_{k=1}^{K_{fs}} C^4_{fks}u_{if}Z_{fksit}$$

$$TC^{\min} = \sum_{f}\sum_{s}\sum_{t}\sum_{k=1}^{K_{fs}} A^1_{fs}Y_{fkst} + \sum_{t}\sum_{f}\sum_{i} H^1_f\left[\sum_{\tau=t-EX_f}^{t}\sum_{s}\sum_{k=1}^{K_{fs}} Z_{fksi\tau} - u_{if}\sum_{\tau=1}^{t} X_{i\tau}\right] + \sum_{t}\sum_{f}\sum_{s}\sum_{i}\sum_{k=1}^{K_{fs}} C^1_{fks}u_{if}Z_{fksit}$$

Where parameter γ is the unit cost of the difference between $TC^{\max}$ and $TC^{\min}$.

Therefore, the robust model of the problem is as described in Equations 2-9.

3. Numerical Findings

In this section, numerical findings and results of numerical calculations of the model are presented. How to prepare the data is explained first, and the results of the problem-solving are then described.

3.1 Problem Data

The algorithm was implemented on three sample problems and the results were determined to evaluate the efficiency of the proposed approach. All problems were implemented on a system with Intel Core i5 CPU, 3.4 GHz, and 8 GB RAM specifications using GAMS software. Table 1 shows the general structure of the sample problems. It should be noted that Problems 1, 2, and 3 have a similar precedence Network (CPM). The beginning and end activities in each problem are imaginary, and the time and resources required for them are zero.

These and hundreds of other problems are created using project scheduling software and are available on the Gantt chart website (http://www.ganttchart.com). Among the problems with various sizes on the website, some were selected in various sizes and used in this study along with a chart of their precedence relationships.

Table 1. The structure of numerical problems

The problem number	The number of activities	The number of perishable resources required
1	22	4
2	22	6
3	22	6

The following is more information about resources and the ordering function (equal for all resources).

Table 2 shows the information on resources.

Table 2. The data of resources

The resource number	**Lifespan (days)**	**The cost of ordering**	**The daily maintenance cost of each inventory**
1	4	[50, 54, 52]	[7, 9, 8]
2	4	[47, 51, 49]	[6, 8, 7]
3	3	[38, 42, 40]	[6, 8, 7]
4	3	[48, 52, 50]	[4, 6, 5]
5	3	[48, 52, 50]	[5, 7, 6]
6	4	[38, 42, 40]	[6, 8, 7]
7	5	[38, 42, 40]	[5, 7, 6]
8	5	[38, 42, 40]	[5, 7, 6]

The information on the total discount function of purchasing the required resources is provided in Table 3.

Table 3. The data of the total discount function of purchasing the required resources

The purchase amount	**The purchase price per unit**
50	20
100	19
200	18
400	17
1000	16
2000	15
4000	14
6000	12

The precedence dependency network of Problems 1 to 3 can be seen in Figure 2. Tables 4 to 7 show the scheduling details for each of the three numerical problems.

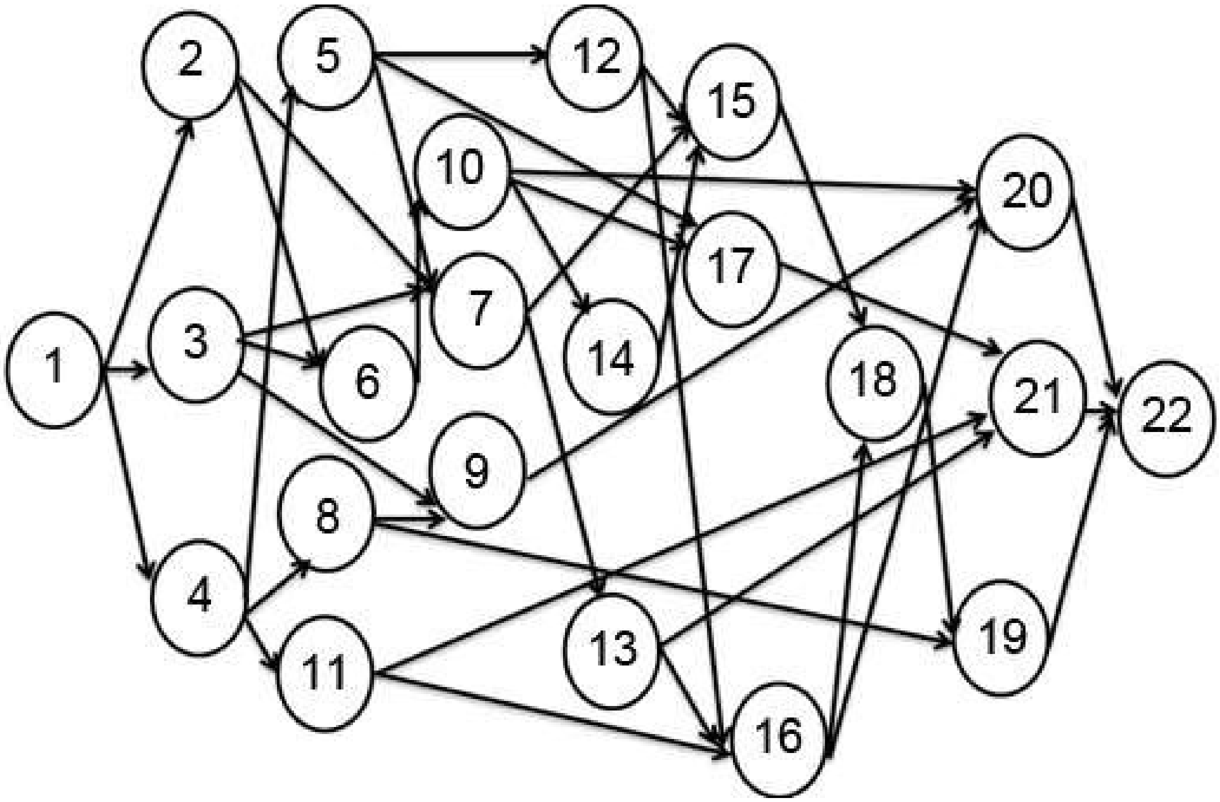

Figure 2. The precedence relationship network of Problems 1 to 3

Table 4. Problem 1 data

Activity	Time	ES	LS	Precedence	R1	R2	R3	R4
1	0	1	1	***	0	0	0	0
2	7	1	3	1	8	4	0	4
3	2	1	8	1	5	8	0	10
4	3	1	2	1	10	10	7	0
5	5	4	5	4	6	8	2	0
6	3	8	11	2, 3	7	4	0	8
7	7	9	10	2, 3, 5	8	7	0	10
8	1	2	24	4	9	8	9	0
9	2	5	25	3, 8	2	5	3	0
10	2	11	14	6	10	9	7	0
11	2	4	19	4	10	9	0	9
12	2	9	16	5	6	7	8	0
13	4	16	17	7	2	9	0	7
14	2	13	16	10	1	8	0	5
15	8	15	18	7, 12, 14	10	4	0	7
16	6	20	21	11, 12, 13	10	5	0	3
17	4	13	22	5, 10	7	5	0	4
18	3	26	26	15, 16	4	6	2	0
19	1	29	29	8, 18	10	6	0	5
20	3	26	27	9, 10, 16	8	2	0	6
21	4	20	26	11, 13, 17	3	6	8	0
22	0	30	30	19, 20, 21	0	0	0	0

Table 5. Problem 2 data

Activity	Time	ES	LS	Precedence	R1	R2	R3	R4	R5	R6
1	0	1	1	***	0	0	0	0	0	0
2	9	1	1	1	8	4	0	3	2	5
3	2	1	8	1	5	5	10	0	4	6
4	4	1	9	1	8	9	7	0	5	0
5	5	5	13	4	7	7	0	9	0	3
6	7	10	10	2, 3	7	4	3	0	5	6
7	10	10	18	2, 3, 5	3	6	0	9	7	2
8	1	5	32	4	7	6	0	7	0	4
9	9	6	33	3, 8	1	5	0	7	6	2
10	8	17	17	6	6	5	0	6	0	4
11	3	5	30	4	8	8	0	7	4	0
12	5	10	26	5	3	5	0	5	6	4
13	5	20	28	7	1	9	0	5	7	8
14	6	25	25	10	1	8	7	0	5	2
15	9	31	31	7, 12, 14	9	4	0	5	6	0
16	7	25	33	11, 12, 13	7	5	6	0	4	2
17	5	25	36	5, 10	5	5	4	0	6	4
18	3	40	40	15, 16	4	6	0	9	9	7
19	3	43	43	8, 18	7	5	8	0	7	4
20	4	32	42	9, 10, 16	8	2	5	0	0	6
21	5	30	41	11, 13, 17	2	5	7	0	1	0
22	0	46	46	19, 20, 21	0	0	0	0	0	0

Table 6. Problem 3 data

Activity	Time	ES	LS	Precedence	R1	R2	R3	R4	R5	R6
1	0	1	1	***	0	0	0	0	0	0
2	10	1	1	1	8	4	0	3	2	5
3	3	1	8	1	5	5	10	0	4	6
4	10	1	7	1	8	9	7	0	5	0
5	6	11	17	4	7	7	0	9	0	3
6	10	11	11	2, 3	7	4	3	0	5	6
7	10	11	23	2, 3, 5	3	6	0	9	7	2
8	6	11	36	4	7	6	0	7	0	4
9	10	17	42	3, 8	1	5	0	7	6	2
10	10	21	21	6	6	5	0	6	0	4
11	5	11	32	4	8	8	0	7	4	0
12	5	17	35	5	3	5	0	5	6	4
13	8	21	33	7	1	9	0	5	7	8
14	9	31	31	10	1	8	7	0	5	2
15	10	40	40	7, 12, 14	9	4	0	5	6	0
16	9	29	41	11, 12, 13	7	5	6	0	4	2
17	8	31	42	5, 10	5	5	4	0	6	4
18	5	50	50	15, 16	4	6	0	9	9	7
19	4	55	55	8, 18	7	5	8	0	7	4
20	7	38	52	9, 10, 16	8	2	5	0	0	6

21	9	39	50	11, 13, 17	2	5	7	0	1	0
22	0	59	59	19, 20, 21	0	0	0	0	0	0

3.2 Numerical Results

The numerical problems presented in the previous section were solved using software, the results of which are shown in Table 7.

Table 7. The time required to solve and the value of the objective function

The problem number	Duration (seconds)	The value of the objective function (*10 ^ 5)
1	174,55	4,54848
2	393,26	8,36956
3	513,18	11,31987

In the following, the findings concerning scheduling and ordering for various numerical problems are given.

3.2.1 Numerical results for Problem 1

Table 8. Optimal scheduling for starting Problem 1 activities

The activity number	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
The day of starting the activity	1	3	8	2	5	11	10	24	25	14	19	16	17	16	18	21	22	26	29	27	26	30

Table 9. Optimal resource ordering policy in Problem 1

Day	The amount of ordering resource 1	The amount of ordering resource 2	The amount of ordering resource 3	The amount of ordering resource 4
1	12	11	5	1
2	21	22	11	3

3	23	20	7	4
4	25	21	9	5
5	29	17	8	4
6	29	37	11	8
7	26	29	5	9
8	22	30	10	14
9	37	38	10	26
10	56	36	15	14
11	42	30	8	31
12	58	66	8	33
13	59	46	16	32
14	50	44	29	53
15	69	81	12	34
16	85	65	19	45
17	80	98	27	42
18	79	57	27	77
19	83	76	17	67
20	126	96	12	49
21	59	80	47	11
22	105	73	14	115
23	141	124	34	56
24	34	156	45	87
25	115	121	60	30
26	41	123	37	104
27	252	254	31	76
28	97	42	59	53
29	71	18	14	77
30	42	34	21	24

3.2.2 Numerical results for Problem 2

Table 10. Optimal scheduling for starting Problem 2 activities

The activity number	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**
The day of starting the activity	1	1	8	9	13	10	18	32	33	17	30	26	28	25	31	33	36	40	43	42	41	46

Table 11. Optimal resource ordering policy in Problem 2

Day	The amount of ordering resource 1	The amount of ordering resource 2	The amount of ordering resource 3	The amount of ordering resource 4	The amount of ordering resource 5	The amount of ordering resource 6
1	16	10	0	6	4	10
2	13	4	0	3	3	8
3	8	4	0	2	2	6
4	5	2	0	4	2	5
5	9	4	0	4	2	4
6	14	8	4	2	3	9
7	25	8	14	1	6	13
8	22	17	15	4	13	18
9	30	33	8	4	12	14
10	29	26	18	2	9	16
11	23	21	30	9	29	20
12	25	22	18	8	12	15
13	42	29	16	14	17	25
14	39	28	15	14	15	18
15	43	27	22	15	11	12
16	42	50	24	10	21	29
17	35	34	28	25	29	30
18	58	59	27	30	20	21
19	34	41	5	27	18	30
20	42	34	14	25	22	30
21	33	23	27	31	28	28
22	53	44	33	12	19	24
23	51	43	15	36	25	19
24	54	63	26	32	38	26
25	48	41	30	16	36	55
26	53	63	14	39	29	36
27	61	86	30	34	36	24
28	32	97	26	26	25	45
29	76	66	31	56	51	28
30	63	61	29	64	76	49
31	27	58	10	39	49	74
32	133	65	39	72	61	24
33	64	90	36	68	40	40
34	84	112	48	10	82	56
35	86	117	38	111	34	55
36	74	67	33	53	83	56
37	90	90	28	62	74	73
38	111	67	30	75	49	35
39	96	104	43	53	115	46
40	16	137	36	88	35	64
41	83	65	92	52	92	84
42	202	236	55	59	49	49
43	112	63	33	102	114	32

44	47	74	46	98	95	99
45	79	55	39	19	67	26
46	14	21	24	33	14	23

3.2.3 **Numerical results for Problem 3**

Table 12. Optimal scheduling for starting Problem 3 activities

The activity number	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
The day of starting the activity	1	1	8	7	17	11	23	36	42	21	32	35	33	31	40	41	42	50	55	52	50	59

Table 13. Optimal resource ordering policy in Problem 3

Day	The amount of ordering resource 1	The amount of ordering resource 2	The amount of ordering resource 3	The amount of ordering resource 4	The amount of ordering resource 5	The amount of ordering resource 6
1	14	9	0	5	4	12
2	9	6	0	4	2	4
3	13	3	0	2	1	4
4	17	8	0	3	2	6
5	6	9	3	3	3	7
6	15	16	4	4	10	9
7	16	12	24	2	8	10
8	20	17	13	3	13	15
9	29	39	11	4	21	14
10	28	11	17	2	9	11
11	34	18	25	4	9	15
12	26	23	18	3	26	15
13	25	21	29	2	8	18
14	43	26	13	3	12	29
15	20	24	24	7	16	8
16	44	44	13	7	17	19
17	19	21	19	20	23	20
18	41	27	18	4	14	25
19	43	38	20	8	12	16
20	52	38	21	24	21	24
21	29	13	23	27	16	29
22	37	28	26	18	26	25
23	35	50	10	29	12	21
24	46	50	24	30	24	28
25	41	47	24	15	30	18
26	46	43	20	35	19	40
27	41	23	24	42	28	26
28	47	35	18	25	18	40
29	47	66	17	12	29	15
30	59	73	9	23	20	24

Day	The amount of ordering resource 1	The amount of ordering resource 2	The amount of ordering resource 3	The amount of ordering resource 4	The amount of ordering resource 5	The amount of ordering resource 6
31	35	49	28	39	34	41
32	51	60	35	44	34	20
33	64	67	28	46	62	41
34	90	80	46	39	41	58
35	58	67	13	56	40	33
36	75	68	42	24	27	34
37	37	76	18	50	54	56
38	35	66	30	65	40	59
39	82	94	15	51	57	29
40	99	81	19	46	81	31
41	61	108	73	51	23	50
42	151	114	22	94	63	81
43	60	89	31	75	55	49
44	70	78	44	44	84	55
45	93	47	25	39	66	51
46	91	134	58	87	74	25
47	84	120	22	93	90	63
48	83	56	41	49	72	55
49	96	70	42	66	38	75
50	89	92	44	103	89	53
51	146	219	52	48	57	71
52	108	125	22	122	106	47
52	46	57	49	19	79	68
54	121	78	63	66	115	72
55	78	77	108	118	25	74
56	121	224	21	57	98	64
57	146	80	34	23	109	60
58	21	13	84	40	24	31
59	12	36	1	49	48	19

4. Conclusion

This study was conducted to provide a robust integrated model for the resource-constrained project scheduling problem (RCPSP) with material ordering under uncertainty. In the proposed model, the problem of robust integrated material ordering and project scheduling was examined in terms of resource constraints, uncertainty, and lag times, assuming that there are several scenarios for the project and the time of performing activities in each scenario was different from other scenarios. The proposed model sought to minimize project costs, including costs of ordering, maintenance, purchase, and penalties for delays in the project delivery minus the bonus for hastening the project delivery. After providing the model, the model robustification was performed with a possibilistic approach. Numerical problems in different dimensions were then designed and solved by using GAMS software, and the results were discussed.

Some of the important points in the model were the simultaneous consideration of non-renewability, perishability, discounts on project resources, uncertainty, lag times, and different project scheduling scenarios. Considering all this in the form of an integrated model of project scheduling and material ordering can lead to a more practical model and its closeness to real-world conditions.

Each study can pave the way for stronger and more comprehensive studies. This study is no exception to this and can be the source of better and stronger studies. According to the results of the study, recommendations for future studies are provided as follows:

- Metaheuristic approaches are recommended to be used to solve large-scale (real-world) problems;
- It is recommended that the proposed model be implemented on a real project as an example and that the results be evaluated;
- Sustainability and environmental issues are recommended to be considered in problem modeling.

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